International Journal of Physics
and Research (IJPR)
ISSN(P): 2250-0030; ISSN(E): 2319-4499
Vol. 4, Issue 3, Jun 2014, 45-52

A NEW CLASS OF BIANCHI TYPE I COSMOLOGICAL MODEL FOR BULK VISCOUS BAROTROPIC FLUID WITH VARIABLE Λ-TERM IN GENERAL RELATIVITY

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ABSTRACT

Exact solution of Einstein's field equations with variable cosmological constant is obtained in presence of bulk viscous barotropic fluid for Bianchi type-I space time. To get a determinate solution of the field equations the average scale factor R^3, in the model is considered as a linear function of t and also the fluid obeys the barotropic equation of state. The physical aspects of the model with astronomical observations are discussed.

KEYWORDS: Cosmology, Exact Solution, Variable Λ, Barotropic Equation of State

1 INTRODUCTION

After the publication of Einstein's geometrical gravitational field equations in 1915, the search for their exact and analytical solutions for all gravitational fields in nature began [1]. General relativistic cosmological models provide a framework for investigation of the evolution of the universe. Present cosmology is based on the Friedmann-Robertson-Walker (FRW) model. In this model, the universe is completely homogeneous and isotropic which is in agreement with the observational data about the large scale of the universe. However there is no reason to believe in a regular expansion for a description of the early stage of the universe. The theoretical arguments [2, 3] and the modern experimental data of the cosmic microwave background radiation which support the existence of an anisotropic phase, which turns into an isotropic one [4]. The choice of anisotropic models in the Einstein system of field equation provides us a systematic way to obtain cosmological model more general than FRW model (1936).

Although the observed universe is almost based on FRW cosmology but it is also believed that in the early universe FRW model does not give a correct matter description. FRW models are unstable near the singularity (Patridge and Wlkinson [5]) and fail to describe early universe. The large scale distribution of galaxies in our universe shows that the matter distribution is satisfactorily described by perfect fluid. But when neutrino decoupling occurred, the matter behaved like viscous fluid in early stage of the universe. Misner [3, 6] have studied the effect of viscosity on the evolution of the universe. Several authors viz. Roy and Prakash [7], Santos *et al.* [8], Gron [9], Ram and Singh [10, 11], Bali *et al.* [12-14] studied the effect of bulk viscosity on the evolution of the universe.

Zel'dovich [15], linde [16] and Dreitlen [17] have studied about the significant of the cosmological constant (Λ) which is a focal point of interest in modern cosmological theories. Cosmological scenarios with a time-varing Λ were proposed by several researchers. In a recent past, a number of authors [18-28] considered cosmological models with time dependent cosmological constant in which Λ decays with time. Bronnikov *et al.* have shown that all the models approach de-Sitter model at late time for small Λ.

Motivated by the above observations, we consider anisotropic space-time of Bianchi type-I model with variable cosmological constant and obtained an exact solution of Einstein's field equations by considering average scale factor R^3 is a linear function of t. This solution is new and different from other authors solutions. The outline of this paper is as follows: in section 2, the metric and field equations are described. Section 3 and 4 deals with the solutions of the field equations and some physical properties of the model respectively. Finally conclusions are summarized in the last section 5.

2 THE METRIC AND FIELD EQUATIONS

We consider Bianchi type-I metric in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 dy^2 + C^2 dz^2 \tag{1}$$

Where A, B and C are functions of t alone

Einstein's field equations with time varying cosmological constant $\Lambda(t)$ is given by

$$R_i^j - \frac{1}{2} g_i^j R = -T_i^j + \Lambda(t) g_i^j \tag{2}$$

Energy momentum tensor T_i^j for bulk viscous fluid is given by

$$T_i^j = (\rho + \bar{p}) \upsilon_i \upsilon^j + \bar{p} g_i^j \tag{3}$$

Where

$$\bar{p} = p - \xi \upsilon_{;i}^i \tag{4}$$

And $\upsilon_i = (0,0,0,-1)$, $\upsilon_i \upsilon^i = -1$, $\upsilon_4 = -1$ and $\upsilon^4 = 1$, p is the isotropic pressure, ρ is the energy density, υ^i is the fluid flow vector.

The scalar curvature for the metric (1) is

$$R = -2\left(\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{C_4 A_4}{CA} + \frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C}\right)$$

Now Einstein's field equations (2) for the metric (1) can be written as

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} = \Lambda - \bar{p} \tag{5a}$$

$$\frac{A_{44}}{A}+\frac{C_{44}}{C}+\frac{A_4C_4}{AC}=\Lambda-\bar{p} \tag{5b}$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}=\Lambda-\bar{p} \tag{5c}$$

$$\frac{A_4B_4}{AB}+\frac{B_4C_4}{BC}+\frac{C_4A_4}{CA}=\Lambda+\rho \tag{5d}$$

For complete determinacy of the system of equations (5), we consider the barotropic equation of state

$$p=w\rho, \quad 0\le w\le 1 \tag{6}$$

The divergence of equation (2) is

$$T_{i;j}^{j}-\Lambda_{;j}g_i^j=0$$

$$\Rightarrow \rho_4+(p+\rho)\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)+\Lambda_4=0 \tag{7}$$

3. SOLUTION OF FIELD EQUATIONS

Equations (5) and (7) are five equations in six unknowns A, B, C, p, ρ and Λ. To find the model of the universe we need one more condition. We have assumed that the average scale factor R^3 is a linear function of t.

Now equations (5a) and (5b) lead to

$$\frac{B_{44}}{B}-\frac{A_{44}}{A}+\frac{C_4}{C}\left(\frac{B_4}{B}-\frac{A_4}{A}\right)=0 \tag{8}$$

From equations (5b) and (5c), we have

$$\frac{C_{44}}{C}-\frac{B_{44}}{B}+\frac{A_4}{A}\left(\frac{C_4}{C}-\frac{B_4}{B}\right)=0 \tag{9}$$

From equation (8), we have

$$\frac{AB_{44}-A_{44}B}{AB_4-A_4B}=-\frac{C_4}{C}$$

$$\Rightarrow \log_e\frac{B}{A}=m\int\frac{dt}{ABC} \tag{10}$$

From equation (9), we have

$$\frac{BC_{44}-B_{44}C}{BC_4-B_4C}=-\frac{A_4}{A}$$

$$\Rightarrow \log_e \frac{C}{B} = n\int \frac{dt}{ABC}$$

$$\Rightarrow \log_e \frac{C}{B} = r\log_e \frac{B}{A}, \text{ where } n = mr$$

$$\Rightarrow B^{r+1} = A^r C \tag{11}$$

Equation (10) implies that

$$\log_e \frac{B}{A} = m\int \frac{dt}{k_1 t + b}$$

$$\Rightarrow B = A(k_1 t + b)^{\frac{m}{k_1}} \tag{12}$$

From equation (111), we have

$$B^{r+2} = A^{r-1}(k_1 t + \lambda)$$

$$\Rightarrow A = (k_1 t + \lambda)^{\frac{1}{3} - \frac{m}{3k_1}(r+2)} \tag{13}$$

Equation (14) implies that

$$B = (k_1 t + \lambda)^{\frac{1}{3} + \frac{m}{3k_1}(1-r)} \tag{14}$$

And from our assumption, we have

$$C = (k_1 t + \lambda)^{\frac{1}{3} + \frac{m}{3k_1}(1+2r)} \tag{15}$$

Using equations (13), (14) and (15) the metric (1) becomes

$$ds^2 = -dt^2 + (k_1 t + \lambda)^{\frac{2}{3} - \frac{2m}{3k_1}(r+2)} dx^2 + (k_1 t + \lambda)^{\frac{2}{3} + \frac{2m}{3k_1}(1-r)} dy^2 + (k_1 t + \lambda)^{\frac{2}{3} + \frac{2m}{3k_1}(1+2r)} dz^2 \tag{16}$$

4 SOME PHYSICAL PROPERTIES OF THE MODEL

The expansion θ is given by

$$\theta = \upsilon^i_{;i} = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}$$

$$\Rightarrow \theta = \frac{k_1}{k_1 t + b} = 3\frac{R_4}{R} = 3H \tag{17}$$

Now

$$\sigma_1^1 = \frac{1}{3}\left(\frac{2A_4}{A} - \frac{B_4}{B} - \frac{C_4}{C}\right)$$

$$= -\frac{1}{3}\frac{m(r+2)}{k_1 t + b} = -\frac{m(r+2)}{3R^3} \tag{18}$$

$$\sigma_2^2 = \frac{1}{3}\left(\frac{2B_4}{B} - \frac{C_4}{C} - \frac{A_4}{A}\right) = \frac{m(1-r)}{3R^3} \tag{19}$$

$$\sigma_3^3 = \frac{1}{3}\left(\frac{2C_4}{C} - \frac{A_4}{A} - \frac{B_4}{B}\right) = \frac{m(1+2r)}{3R^3} \tag{20}$$

$$\sigma_4^4 = 0$$

Now

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij}$$

$$= \frac{1}{2}\left\{(\sigma_1^1)^2 + (\sigma_2^2)^2 + (\sigma_3^3)^2 + (\sigma_4^4)^2\right\} = \frac{m^2(r^2+r+1)}{3R^6} \tag{21}$$

Deceleration parameter q is given by

$$q = -\frac{R_{44}}{RH} = -\frac{R_{44}/R}{R_4{}^2/R^2} = 2 \tag{22}$$

From equation (5c), we have

$$-\frac{3k_1{}^2}{9(k_1 t + b)^2} + \frac{m^2(r^2+r+1)}{3(k_1 t + b)^2} = \Lambda - \bar{p}$$

$$\Rightarrow \Lambda - \bar{p} = -H^2(2q-1) + \sigma^2 \tag{23}$$

Again from equation (5d), we have

$$\frac{3k_1{}^2}{9(k_1 t + b)^2} - \frac{m^2(r^2+r+1)}{3(k_1 t + b)^2} = \Lambda + \rho$$

$$\Rightarrow \Lambda + \rho = 3H^2 - \sigma^2 \tag{24}$$

Equations (25) and (26) lead to

$$\rho = \frac{1}{(1+w)}\left\{2(q+1)H^2 - 2\sigma^2 + \xi\theta\right\} \tag{25}$$

Equation (26) leads to

$$\Lambda = \frac{1}{(1+w)}\left\{(3w-2q+1)H^2 - (w-1)\sigma^2 - \xi\theta\right\} \tag{26}$$

Equation (23) leads to

$$p = \frac{w}{(1+w)}\left\{2(1+2q)H^2 - 2\sigma^2 + \xi\theta\right\} \tag{27}$$

Now since

$$\int_{t_0}^{t} \frac{dt}{V(t)} = \int_{t_0}^{t} \frac{dt}{(k_1 t + b)^{1/3}} = \frac{3}{2k_1}\left[(k_1 t + b)^{2/3}\right]_{t_0}^{t}$$

Which is a convergent integral, so the particle horizon exists

From the above result, it can be seen that the spatial volume is zero at $t = t_0$ where $t_0 = \lambda/k_1$. At this epoch the energy density is infinite. The physical quantities p and ρ, the scalar of expansion θ, shear scalar σ^2, Hubble's parameter H and the cosmological constant Λ are diverse at $t = t_0$ while they vanish for large value of t. Equation is a convergent integral, so the particle horizon exists.

5 CONCLUSIONS

In this paper we have presented a new solution of Einstein's field equations for anisotropic Bianchi type-I space-time in presence of bulk viscous fluid with time varying cosmological constant. The model starts with a big-bank at $t = t_0$ and goes on expanding until it comes out rest at $t = \infty$. The model asymptotically tends to a de-Sitter universe for large value of t. It is observed that all physical parameters are infinite at the initial epoch $t = t_0$ and tends to zero for large t. Spatial volume increase as time increases. As $\lim_{t\to\infty}\frac{\sigma}{\theta} \neq 0$, hence the model does not approach isotropy for large time.

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